

3. Consider the function $f(x, y, z) = 3xy + z^3$ and the vector $\mathbf{A} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

a. Compute ∇f .

$$\vec{\nabla} f = f_x \vec{i} + f_y \vec{j} + f_z \vec{k} = 3y \vec{i} + 3x \vec{j} + 3z^2 \vec{k}$$

The set of points $P(x, y, z)$ where f increases fastest in the direction of $\mathbf{A}$ is a curve $\mathcal{C}$.

b. Express the curve $\mathcal{C}$ as an intersection $\mathcal{S}_1 \cap \mathcal{S}_2$ of two cylinders $\mathcal{S}_1$ and $\mathcal{S}_2$ by finding an equation for each of these cylinders.

f increases fastest in the direction of $\vec{A} \iff \vec{\nabla} f = c \cdot \vec{A}$ for some $c \geq 0$

$$\iff \frac{3y}{2} = \frac{3x}{1} = \frac{3z^2}{2}$$

We can take $\mathcal{S}_1: x = \frac{1}{2}z^2$ and $\mathcal{S}_2: y = z^2$

c. Find a differentiable parametrization of the curve $\mathcal{C}$.

$$\vec{r} = \frac{1}{2}t^2 \vec{i} + t^2 \vec{j} + t \vec{k}, \quad (-\infty < t < \infty)$$

d. Find parametric equations for the tangent line to the curve $\mathcal{C}$ at the point $P_0(1/8, 1/4, 1/2)$ by using either ☐ Part b or ☒ Part c. [Indicate your choice by putting a **X** in the corresponding ☐.

$$\vec{v} = \frac{d\vec{r}}{dt} = t \vec{i} + 2t \vec{j} + \vec{k} \Rightarrow \vec{v} \Big|_{t=\frac{1}{2}} = \frac{1}{2} \vec{i} + \vec{j} + \vec{k}$$

$$\left. \begin{aligned} x &= \frac{1}{2}t + \frac{1}{8} \\ y &= t + \frac{1}{4} \\ z &= t + \frac{1}{2} \end{aligned} \right\} (-\infty < t < \infty)$$