

2. Evaluate the following limits.

a. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y^8}{x^8 + y^{12}}$

$$\left. \begin{array}{l} a=3, b=8 \\ c=8, d=12 \end{array} \right\} \Rightarrow \frac{a}{c} + \frac{b}{d} = \frac{3}{8} + \frac{8}{12} = \frac{25}{24} > 1$$

$$\Rightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y^8}{x^8 + y^{12}} = 0 \quad \text{by L'Hôpital's Theorem.}$$

b. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^9}{x^8 + y^{12}}$

$$\left. \begin{array}{l} a=2, b=9 \\ c=8, d=12 \end{array} \right\} \Rightarrow \frac{a}{c} + \frac{b}{d} = \frac{2}{8} + \frac{9}{12} = 1 \leq 1$$

$$\Rightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^9}{x^8 + y^{12}} \text{ does not exist by L'Hôpital's Theorem.}$$

c. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^7 y^2}{x^8 - x^5 y^5 + y^{12}} = \lim_{(x,y) \rightarrow (0,0)} \frac{\frac{x^7 y^2}{x^8 + y^{12}}}{1 - \frac{x^5 y^5}{x^8 + y^{12}}} = \frac{0}{1-0} = 0 \quad \text{as}$

$$\left. \begin{array}{l} a=7, b=2 \\ c=8, d=12 \end{array} \right\} \Rightarrow \frac{a}{c} + \frac{b}{d} = \frac{7}{8} + \frac{2}{12} = \frac{25}{24} > 1 \Rightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{x^7 y^2}{x^8 + y^{12}} = 0$$

and

$$\left. \begin{array}{l} a=5, b=5 \\ c=8, d=12 \end{array} \right\} \Rightarrow \frac{a}{c} + \frac{b}{d} = \frac{5}{8} + \frac{5}{12} = \frac{25}{24} > 1 \Rightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{x^5 y^5}{x^8 + y^{12}} = 0$$

by L'Hôpital's Theorem.