

2. Determine whether each of the following series converges or diverges.

a. $\sum_{n=1}^{\infty} \left(\frac{2n-1}{2n+1} \right)^n$

$$\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+1} \right)^n = \lim_{n \rightarrow \infty} \frac{\left(1 - \frac{1/2}{n} \right)^n}{\left(1 + \frac{1/2}{n} \right)^n} = \frac{e^{-1/2}}{e^{1/2}} = e^{-1} \neq 0$$

by "useful limits"

$\Rightarrow \sum_{n=1}^{\infty} \left(\frac{2n-1}{2n+1} \right)^n$ diverges by n^{th} Term Test.

b. $\sum_{n=1}^{\infty} \frac{\sin(5n)}{n^2 + \sin^2(3n)}$

$$0 < n^2 \leq n^2 + \sin^2(3n) \text{ and } 0 \leq |\sin(5n)| \leq 1 \text{ for } n \geq 1$$

$$\Rightarrow 0 < \frac{|\sin(5n)|}{n^2 + \sin^2(3n)} \leq \frac{1}{n^2} \text{ for } n \geq 1$$

Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p -series with $p=2 > 1$),

$\sum_{n=1}^{\infty} \frac{|\sin(5n)|}{n^2 + \sin^2(3n)}$ converges by Direct Comparison Test.

$\Downarrow$

$\sum_{n=1}^{\infty} \frac{\sin(5n)}{n^2 + \sin^2(3n)}$ converges by Absolute Convergence Test.