

2. Determine all values of the positive constant a for which

$$\iint_{D(a)} (y - x^2) dA = 0$$

where $D(a)$ is the intersection of the disk with radius a and center at the origin with the first quadrant.

$$\iint_{D(a)} (y - x^2) dA = \int_0^{\pi/2} \int_0^a (r \sin \theta - r^2 \cos^2 \theta) r dr d\theta$$

$$= \int_0^{\pi/2} \left[\frac{1}{3} r^3 \sin \theta - \frac{1}{4} r^4 \cos^2 \theta \right]_{r=0}^{r=a} d\theta$$

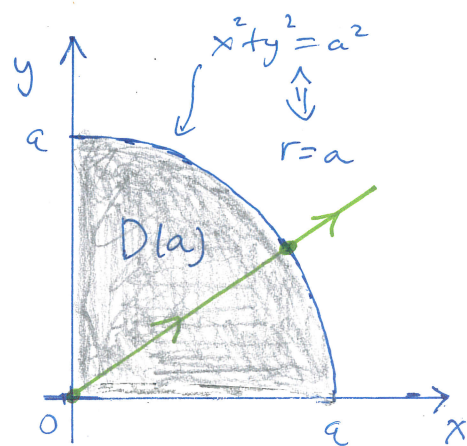
$$= \frac{a^3}{3} \int_0^{\pi/2} \sin \theta d\theta - \frac{a^4}{4} \int_0^{\pi/2} \cos^2 \theta d\theta$$

$$= \frac{a^3}{3} [-\cos \theta]_0^{\pi/2} - \frac{a^4}{4} \int_0^{\pi/2} \frac{1 + \cos 2\theta}{2} d\theta$$

$$= \frac{a^3}{3} \cdot \left(\underbrace{-\cos \frac{\pi}{2}}_0 + \underbrace{\cos 0}_1 \right) - \frac{a^4}{8} \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi/2}$$

$$= \frac{a^3}{3} - \frac{a^4}{8} \left(\left(\frac{\pi}{2} + \frac{\sin \pi}{2} \right) - \left(0 + \frac{\sin 0}{2} \right) \right)$$

$$= \frac{a^3}{3} - \frac{\pi a^4}{16}$$



$$\iint_{D(a)} (y - x^2) dA = 0 \Leftrightarrow \frac{a^3}{3} - \frac{\pi a^4}{16} = 0 \Leftrightarrow a = \frac{16}{3\pi}$$

$D(a)$

for $a > 0$