

1. Consider the radial diffusion equation

$$\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial r^2} + \frac{a}{r} \cdot \frac{\partial f}{\partial r}$$

where  $f(r, t)$  is a function of the distance to the origin  $r$  and the time  $t$ , and  $a$  is a constant depending on the dimension of the space.

Find all values of the pair of constants  $(a, b)$  for which the function

$$f(r, t) = \frac{e^{-br^2/t}}{t^3}$$

satisfies the radial diffusion equation for all  $(r, t)$  with  $r > 0$  and  $t > 0$ .

$$f(r, t) = e^{-br^2 t^{-1}} \cdot t^{-3}$$

$$f_t = e^{-br^2 t^{-1}} \cdot (br^2 t^{-2}) \cdot t^{-3} + e^{-br^2 t^{-1}} \cdot (-3t^{-4})$$

$$f_r = e^{-br^2 t^{-1}} \cdot (-2br t^{-1}) \cdot t^{-3} = e^{-br^2 t^{-1}} \cdot (-2br t^{-4})$$

$$f_{rr} = e^{-br^2 t^{-1}} \cdot (-2br t^{-1})(-2br t^{-4}) + e^{-br^2 t^{-1}} \cdot (-2bt^{-4})$$

$$f_t = f_{rr} + \frac{a}{r} f_r$$

$$\begin{aligned} &\Downarrow \\ e^{-br^2 t^{-1}} (br^2 t^{-5} - 3t^{-4}) &= e^{-br^2 t^{-1}} (4b^2 r^2 t^{-5} - 2bt^{-4} - 2abt^{-4}) \\ &\quad \text{for all } r > 0, t > 0 \end{aligned}$$

$\Downarrow$

$$(b - 4b^2)r^2 + (2b(1+a) - 3)t = 0 \quad \text{for all } r > 0, t > 0$$

$\Downarrow$

$$\begin{aligned} &b - 4b^2 = 0 \quad \text{or} \quad b = 0 \quad \text{or} \quad b = \frac{1}{4} \\ &2b(1+a) - 3 = 0 \quad \text{or} \quad a = 5 \end{aligned}$$

$0 = 3 \otimes$

$(a, b) = (5, \frac{1}{4})$  are the only values satisfying the condition.