

3. In each of the following, if the given statement is true for all sequences $\{a_n\}_{n=1}^{\infty}$, then mark the $\square$ to the left of TRUE with a **X**; otherwise, mark the $\square$ to the left of FALSE with a **X** and give a counterexample. No explanation is required.

a. If $\lim_{n \rightarrow \infty} a_n$ exists, then $\sum_{n=1}^{\infty} a_n$ converges.

☐ TRUE

☒ FALSE, because it does not hold for $a_n =$ 1 for $n \geq 1$

b. If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \rightarrow \infty} a_n$ exists.

☒ TRUE

☐ FALSE, because it does not hold for $a_n =$ for $n \geq 1$

c. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n = \lim_{n \rightarrow \infty} a_n$.

☐ TRUE

☒ FALSE, because it does not hold for $a_n =$ $\frac{1}{2^n}$ for $n \geq 1$

d. If $\sum_{n=1}^{\infty} a_n$ converges, then the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ is 1.

☐ TRUE

☒ FALSE, because it does not hold for $a_n =$ 0 for $n \geq 1$

e. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n x^n$ converges absolutely for $|x| < 1$.

☒ TRUE

☐ FALSE, because it does not hold for $a_n =$ for $n \geq 1$