

1. Find the absolute maximum and minimum values of $f(x, y) = \frac{1}{4}y^4 - (y - 2x)^2$ on the rectangle $D = \{(x, y) : -1 \leq x \leq 1 \text{ and } -1 \leq y \leq 3\}$.

Interior of D: $f_x = -2(y - 2x) \cdot (-2) = 0 \Rightarrow y = 2x \xrightarrow{x=0}$
 $f_y = y^3 - 2(y - 2x) = 0 \xrightarrow{y^3=0 \Rightarrow y=0} \Rightarrow$
Critical point: $(x, y) = \underline{(0, 0)}$

Boundary of D:

Side 1: $y = -1$ and $-1 \leq x \leq 1 \Rightarrow f(x, -1) = \frac{1}{4} - (2x + 1)^2$ for $-1 \leq x \leq 1$

Critical points: $\frac{d}{dx} f(x, -1) = -2(2x + 1) \cdot 2 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow (x, y) = \underline{(-\frac{1}{2}, -1)}$

Endpoints: $x = -1, 1 \Rightarrow (x, y) = \underline{(-1, -1)}, \underline{(1, -1)}$

Side 2: $x = -1$ and $-1 \leq y \leq 3 \Rightarrow f(-1, y) = \frac{1}{4}y^4 - (y + 2)^2$ for $-1 \leq y \leq 3$

Critical points: $\frac{d}{dy} f(-1, y) = y^3 - 2(y + 2) = (y - 2)(y^2 + 2y + 2) = 0 \Rightarrow y = 2 \Rightarrow (x, y) = \underline{(-1, 2)}$

Endpoints: $y = -1, 3 \Rightarrow (x, y) = \underline{(-1, -1)}, \underline{(-1, 3)}$

Side 3: $y = 3$ and $-1 \leq x \leq 1 \Rightarrow f(x, 3) = \frac{81}{4} - (2x - 3)^2$ for $-1 \leq x \leq 1$

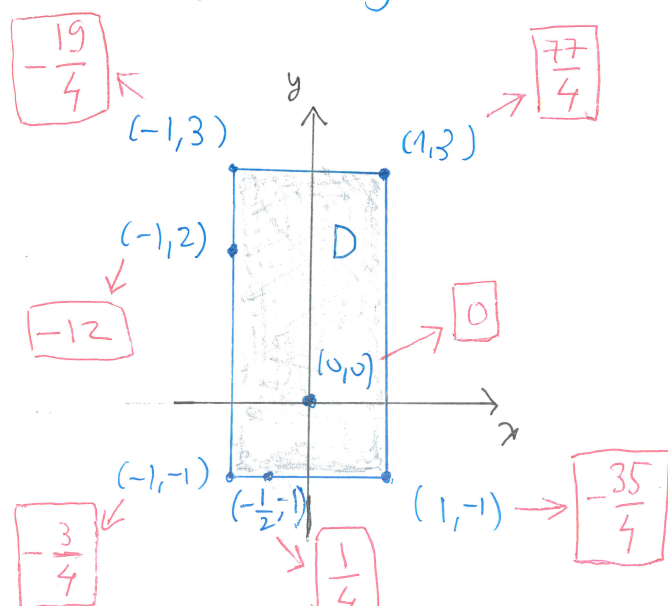
Critical points: $\frac{d}{dx} f(x, 3) = -2(2x - 3) \cdot 2 = 0 \Rightarrow x = \frac{3}{2} \leftarrow \text{not in the interval!}$

Endpoints: $x = -1, 1 \Rightarrow (x, y) = \underline{(-1, 3)}, \underline{(1, 3)}$

Side 4: $x = 1$ and $-1 \leq y \leq 3 \Rightarrow f(1, y) = \frac{1}{4}y^4 - (y - 2)^2$ for $-1 \leq y \leq 3$

Critical points: $\frac{d}{dy} f(1, y) = y^3 - 2(y - 2) = (y + 2)(y^2 - 2y + 2) = 0 \Rightarrow y = -2 \leftarrow \text{not in the interval!}$

Endpoints: $y = -1, 3 \Rightarrow (x, y) = \underline{(1, -1)}, \underline{(1, 3)}$



Absolute maximum is $\frac{77}{4}$.

Absolute minimum is -12 .