

2. Evaluate the following limits.

a. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^7}{x^6 + y^{10}} = 0$

$$\frac{a}{c} + \frac{b}{d} = \frac{2}{6} + \frac{7}{10} = \frac{31}{30} > 1 \Rightarrow \text{The limit is 0 by Sertöz Theorem}$$

b. $\lim_{(x,y) \rightarrow (0,0)} \frac{xy(x-y)}{x^2+y^2} = \underbrace{\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2+y^2}}_{(1)} - \underbrace{\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2+y^2}}_{(2)} = 0 - 0 = 0$

$$\frac{a}{c} + \frac{b}{d} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2} > 1 \Rightarrow (1) \text{ is 0 by Sertöz Theorem.}$$

$$\frac{a}{c} + \frac{b}{d} = \frac{1}{2} + \frac{2}{2} = \frac{3}{2} > 1 \Rightarrow (2) \text{ is 0 by Sertöz Theorem.}$$

c. $\lim_{(x,y) \rightarrow (0,0)} \frac{xy(x-y)^2}{x^4+y^4}$

$\lim_{(x,y) \rightarrow (0,0)} \frac{xy \cdot (x-y)^2}{x^4+y^4} = \lim_{x \rightarrow 0} \frac{x \cdot 0 \cdot (x-0)^2}{x^4+0^4} = \lim_{x \rightarrow 0} \frac{0}{x^4} = \lim_{x \rightarrow 0} 0 = 0$
along the x-axis

$\lim_{(x,y) \rightarrow (0,0)} \frac{xy \cdot (x-y)^2}{x^4+y^4} = \lim_{x \rightarrow 0} \frac{x \cdot (-x) \cdot (x-(-x))^2}{x^4+(-x)^4} = \lim_{x \rightarrow 0} \frac{-4x^4}{2x^4} = \lim_{x \rightarrow 0} (-2) = -2$
along the line $y = -x$

$\Rightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{xy(x-y)^2}{x^4+y^4}$ does not exist by 2-Path Test.