

3. Let  $E = \{(x, y, z) : x^2 + y^2 + z^2 \leq 2z \text{ and } x^2 + y^2 + z^2 \leq 1 \text{ and } 0 \leq x \leq y\}$ .

a. Fill in the boxes so that the following equality holds for all continuous functions  $f$  where  $(r, \theta, z)$  are the cylindrical coordinates.

$$\iiint_E f(x, y, z) dV = \int_{\boxed{\pi/4}}^{\boxed{\pi/2}} \int_{\boxed{0}}^{\boxed{\sqrt{3}/2}} \int_{\boxed{1-\sqrt{1-r^2}}}^{\boxed{\sqrt{1-r^2}}} f(r \cos \theta, r \sin \theta, z) r dz dr d\theta$$

b. Fill in the boxes so that the following equality holds for all continuous functions  $f$  where  $(\rho, \phi, \theta)$  are the spherical coordinates.

$$\iiint_E f(x, y, z) dV$$

$$= \int_{\boxed{\pi/4}}^{\boxed{\pi/2}} \int_{\boxed{\pi/3}}^{\boxed{\pi/2}} \int_{\boxed{0}}^{\boxed{2 \cos \phi}} f(\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi) \rho^2 \sin \phi d\rho d\phi d\theta$$

$$+ \int_{\boxed{\pi/4}}^{\boxed{\pi/2}} \int_{\boxed{0}}^{\boxed{\pi/3}} \int_{\boxed{0}}^{\boxed{1}} f(\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi) \rho^2 \sin \phi d\rho d\phi d\theta$$

