

2. Evaluate the following integrals.

a. $\iint_D x^3 e^{x^2 y} dA$ where $D = \{(x, y) : 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 1/2\}$

$$\begin{aligned} \iint_D x^3 e^{x^2 y} dA &= \int_0^2 \int_0^{1/2} x^3 e^{x^2 y} dy dx = \int_0^2 x e^{x^2 y} \Big|_{y=0}^{y=1/2} dx \\ &= \int_0^2 (x e^{x^2/2} - x) dx = \left[e^{x^2/2} - \frac{1}{2} x^2 \right]_0^2 = e^2 - 2 - (1 - 0) = e^2 - 3 \end{aligned}$$

b. $\int_0^{\pi/2} \int_{2x/\pi}^1 \cos(x/y) dy dx = \iint_D \cos(x/y) dA = \int_0^1 \int_0^{\pi y/2} \cos(x/y) dx dy$

$$\begin{aligned} &= \int_0^1 y \sin(x/y) \Big|_{x=0}^{x=\pi y/2} dy = \int_0^1 (y \sin(\pi/2) - y \sin(0)) dy = \int_0^1 y dy = \frac{1}{2} y^2 \Big|_0^1 \\ &= \frac{1}{2} - 0 = \frac{1}{2} \end{aligned}$$

