

1. Consider the function:

$$f(x, y) = 8x + 2xy - xy^2 - 3x^2$$

The complete list of the critical points and the values of f at these points is given in the table. [You do not need to verify these.]

(a, b)	$(0, -2)$	$(0, 4)$	$(3/2, 1)$
$f(a, b)$	0	0	27/4

Find the absolute maximum and minimum values of f on $D = \{(x, y) : -1 \leq x \leq 2 \text{ and } 0 \leq y \leq 3\}$.

Boundary of D :

Side 1: $y=0$ and $-1 \leq x \leq 2 \Rightarrow f(x, 0) = 8x - 3x^2$ for $-1 \leq x \leq 2$

Critical points: $\frac{d}{dx} f(x, 0) = 8 - 6x = 0 \Rightarrow x = \frac{4}{3} \Rightarrow (x, y) = (\frac{4}{3}, 0)$

Endpoints: $x = -1, x = 2 \Rightarrow (x, y) = (-1, 0), (2, 0)$

Side 2: $y=3$ and $-1 \leq x \leq 2 \Rightarrow f(x, 3) = 5x - 3x^2$ for $-1 \leq x \leq 2$

Critical points: $\frac{d}{dx} f(x, 3) = 5 - 6x = 0 \Rightarrow x = \frac{5}{6} \Rightarrow (x, y) = (\frac{5}{6}, 3)$

Endpoints: $x = -1, x = 2 \Rightarrow (x, y) = (-1, 3), (2, 3)$

Side 3: $x=-1$ and $0 \leq y \leq 3 \Rightarrow f(-1, y) = y^2 - 2y - 11$ for $0 \leq y \leq 3$

Critical points: $\frac{d}{dy} f(-1, y) = 2y - 2 = 0 \Rightarrow y = 1 \Rightarrow (x, y) = (-1, 1)$

Endpoints: $y=0, y=3 \Rightarrow (x, y) = (-1, 0), (-1, 3)$

Side 4: $x=2$ and $0 \leq y \leq 3 \Rightarrow f(2, y) = -2y^2 + 4y + 4$ for $0 \leq y \leq 3$

Critical points: $\frac{d}{dy} f(2, y) = -4y + 4 = 0 \Rightarrow y = 1 \Rightarrow (x, y) = (2, 1)$

Endpoints: $y=0, y=3 \Rightarrow (x, y) = (2, 0), (2, 3)$

Interior of D :

$(\frac{3}{2}, 1)$ is the only critical point in the interior of D

Absolute maximum is $\frac{27}{4}$.

Absolute minimum is -12 .

