4. Find an equation for the plane passing through the point $P_0(3,2,1)$ and containing the line of intersection of the planes $\mathscr{P}_1 : x+y+z=3$ and $\mathscr{P}_2 : x+2y+3z=6$.

① ②

Consider the equation $a \times ① + b \times ②$ where a, b are constants, not both 0:

$$\mathscr{P}: \quad a\cdot(x+y+z) + b\cdot(x+2y+3z) = 3a+6b \qquad ③$$

This equation defines a plane in space.
Since every point (x,y,z) which satisfies both equation ① and equation ② also satisfies equation ③, this plane contains the line of intersection of $\mathscr{P}_1$ and $\mathscr{P}_2$.

$\mathscr{P}$ contains the point P_0 exactly when $a\cdot(3+2+1)+b\cdot(3+2\cdot2+3\cdot1)=3a+6b$.

$$\Updownarrow$$

$$3a+4b=0$$

Let us choose $a=4$ and $b=-3$. Then

$$\mathscr{P}: \ x-2y-5z=-6$$

is an equation for the plane we seek.