

2. An artist is designing a sculpture consisting of a solid sphere sitting on top of a solid cube. The total height of the sculpture will be 1 m. The material used to construct the solid sphere costs twice as much per unit volume as the material used to construct the solid cube. Determine the side length of the cube for (a) the most expensive and (b) the least expensive versions of this sculpture.

$$\text{Cost} = x^3 + 2 \cdot \frac{4\pi}{3} \cdot \left(\frac{1-x}{2}\right)^3 \quad \text{for } 0 \leq x \leq 1$$

Maximize/Minimize $\text{Cost} = x^3 + \frac{\pi}{3} (1-x)^3$ for $0 \leq x \leq 1$.

Critical points: $\frac{d\text{Cost}}{dx} = 3x^2 + \pi(1-x)^2(-1) = 0 \Rightarrow x = \frac{1}{1 \pm \sqrt{\frac{3}{\pi}}} \text{ m}$

$x = \frac{1}{1 - \sqrt{\frac{3}{\pi}}} > 1$ is not in the interval.

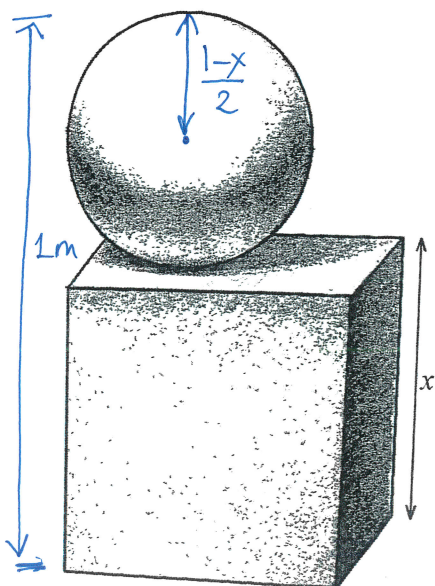
Only critical point in the interval is $x = \frac{1}{1 + \sqrt{\frac{3}{\pi}}} \text{ m}$

$$\Rightarrow \text{Cost} = \frac{1}{\left(1 + \sqrt{\frac{3}{\pi}}\right)^2}$$

Endpoints:

$$x = 0 \text{ m} \Rightarrow \text{Cost} = \frac{\pi}{3}$$

$$x = 1 \text{ m} \Rightarrow \text{Cost} = 1$$



In the most expensive sculpture
the side length of the cube is 0 m.

In the least expensive sculpture
the side length of the cube is $\frac{1}{1 + \sqrt{\frac{3}{\pi}}} \text{ m}$.