

1. Sketch the graph of $y = x^{2/3} + 2x^{-1/3}$ by computing y' and y'' , and determining their signs and the corresponding shapes; finding the critical points, the inflections points, the intercepts, and the asymptotes; and clearly labeling them in the picture.

$$y' = \frac{2}{3} x^{-1/3} - \frac{2}{3} x^{-4/3} = \frac{2}{3} x^{-4/3} \cdot (x-1) = 0 \Rightarrow x=1$$

$\nwarrow \quad \nearrow$
 $x=0$

$$y'' = -\frac{2}{9} x^{-4/3} + \frac{8}{9} x^{-7/3} = -\frac{2}{9} x^{-7/3} \cdot (x-4) = 0 \Rightarrow x=4$$

x	0	1	4	
y'	-	-	+	+
y''	-	+	+	-

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$$x=1 \Rightarrow y=3$$

$$x=4 \Rightarrow y = 3 \cdot 2^{1/3}$$

$$\lim_{x \rightarrow 0^+} y = \infty, \quad \lim_{x \rightarrow 0^-} y = -\infty$$

$$y=0 \Rightarrow x^{-1/3} \cdot (x+2) = 0 \Rightarrow x=-2$$

$$y = x^{2/3} + 2x^{-1/3}$$

