

4. A ship, whose speed and direction are changing all the time in a stormy sea, is being observed by two lighthouses A and B that lie 3 km apart on a straight shoreline. At a certain moment,

- the distance of the ship to lighthouse A is 5 km and increasing at a rate of 10 km/h, and
- the distance of the ship to lighthouse B is 7 km and increasing at a rate of 20 km/h.

Determine how fast the distance from the ship to the shore is changing at this same moment.

[Do not assume anything about how the quantities in the question depend on time beyond what is already given in the question.]

Let a, b, h, x be as shown in the figure.

At our moment, $a = 7$ km, $\frac{da}{dt} = 20$ km/h, $b = 5$ km, $\frac{db}{dt} = 10$ km/h.

$$\begin{aligned} (1) \quad x^2 + h^2 &= b^2 \\ (2) \quad (3-x)^2 + h^2 &= a^2 \end{aligned} \quad \begin{aligned} \Rightarrow x^2 + h^2 &= 5^2 \\ \Rightarrow (3-x)^2 + h^2 &= 7^2 \end{aligned} \quad \begin{aligned} & \xrightarrow{\text{subtract}} \\ & -9 + 6x = -24 \Rightarrow x = -\frac{5}{2} \text{ km} \end{aligned} \quad \left. \begin{aligned} & \xrightarrow{\text{substitute into (1)}} \\ & h = \frac{5\sqrt{3}}{2} \text{ km} \end{aligned} \right\}$$

at our moment

$$\begin{aligned} (1) \Rightarrow x \frac{dx}{dt} + h \frac{dh}{dt} &= b \frac{db}{dt} \quad \Rightarrow -\frac{5}{2} \cdot \frac{dx}{dt} + \frac{5\sqrt{3}}{2} \cdot \frac{dh}{dt} = 5 \cdot 10 \quad (3) \\ (2) \Rightarrow -(3-x) \frac{dx}{dt} + h \frac{dh}{dt} &= a \frac{da}{dt} \quad \Rightarrow -\frac{11}{2} \cdot \frac{dx}{dt} + \frac{5\sqrt{3}}{2} \cdot \frac{dh}{dt} = 7 \cdot 20 \quad (4) \end{aligned}$$

at our moment

$$-11 \times (3) + 5 \times (4) \Rightarrow \frac{5\sqrt{3}}{2} \cdot (-11 + 5) \cdot \frac{dh}{dt} = -11 \cdot 5 \cdot 10 + 5 \cdot 7 \cdot 20 \Rightarrow \frac{dh}{dt} = -\frac{10}{\sqrt{3}} \text{ km/h}$$

The distance from the ship to the shore is decreasing

at a rate of $\frac{10}{\sqrt{3}}$ km/h at that moment.

