

2. A differentiable function f satisfies $f(x) + f(\sqrt{x}) = x$ for all $x > 0$.

a. Find $f(1)$.

$$x=1 \xrightarrow{\oplus} f(1) + f(1) = 1 \Rightarrow f(1) = \frac{1}{2}$$

b. Find $f'(1)$.

$$\begin{aligned} \textcircled{*} \xrightarrow{d/dx} f'(x) + f'(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} &= 1 \xrightarrow{x=1} f'(1) + f'(1) \cdot \frac{1}{2} = 1 \\ &\Downarrow \\ f'(1) &= \frac{2}{3} \end{aligned}$$

c. Write an equation for the tangent line to the graph of $y = f(x)$ at the point with $x = 1$.

$$y - \frac{1}{2} = \frac{2}{3} \cdot (x - 1)$$

 Write only the equation of the line
(and nothing else) in the box!

d. Estimate the value of $f(3/4)$ using the linear approximation of f centered at $x = 1$.

$$L(x) = \frac{1}{2} + \frac{2}{3} \cdot (x - 1)$$

$$\Rightarrow f(3/4) \approx L(3/4) = \frac{1}{2} + \frac{2}{3} \cdot \left(\frac{3}{4} - 1\right) = \frac{1}{3}$$