

1. Consider the function $f(x) = \frac{\sin 3x}{x}$.

a. Compute $f'(x)$.

[Simplify!]

$$f'(x) = \frac{3 \cos 3x \cdot x - \sin 3x \cdot 1}{x^2} = 3 \frac{\cos 3x}{x} - \frac{\sin 3x}{x^2}$$

b. Compute $f''(x)$.

[Simplify!]

$$\begin{aligned} f''(x) &= 3 \frac{-3 \sin 3x \cdot x - \cos 3x \cdot 1}{x^2} - \frac{3 \cos 3x \cdot x^2 - \sin 3x \cdot 2x}{x^4} \\ &= -9 \frac{\sin 3x}{x} - 6 \frac{\cos 3x}{x^2} + 2 \frac{\sin 3x}{x^3} \end{aligned}$$

c. Find constants a and b such that $f''(x) + \frac{a}{x}f'(x) + bf(x) = 0$ for all $x \neq 0$.

For $a=2$ and $b=9$, $f''(x) + \frac{2}{x}f'(x) + 9f(x) = 0$ for all $x \neq 0$.

d. Show that if c is a critical point of f , then $f''(c)f(c) < 0$.

c is a critical point $\Rightarrow f'(c) = 0 \Rightarrow f''(c) = -9f(c)$ by Part c
 $\Rightarrow f''(c)f(c) = -9f(c)^2 \Rightarrow f''(c)f(c) < 0$ or $f(c) = 0$.

$\left. \begin{array}{l} f(c) = 0 \Rightarrow \sin 3c = 0 \\ f'(c) = 0 \text{ and } f(c) = 0 \Rightarrow \cos 3c = 0 \end{array} \right\} \Rightarrow 0 = \sin^2 3c + \cos^2 3c = 1$
 a contradiction.
 So $f(c)$ cannot be 0.