

4. In each of the following, if there exist any polynomials $P(x)$ that satisfy the given condition, then give an example of such a polynomial of the smallest degree; otherwise just write DOES NOT EXIST inside the box. No explanation is required. No partial credit will be given.

- a. The line $y = \frac{1}{2}x + \frac{3}{4}$ is a slant asymptote of the graph of $y = \frac{P(x)}{P'(x)}$.

$$P(x) = x^2 + 3x$$

- b. The line $y = \frac{3}{4}x + \frac{1}{2}$ is a slant asymptote of the graph of $y = \frac{P(x)}{P'(x)}$.

$$P(x) = \text{DNE}$$

- c. $P(x)$ has exactly two critical points: one at $x = -2$ and another one at $x = 1$.

$$P(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x$$

- d. $P(x)$ has exactly three critical points and exactly two real roots.

$$P(x) = x^4 - x^2 - 1$$

- e. The improper integral $\int_{-\infty}^{\infty} P(x)^{-1/3} dx$ is convergent.

$$P(x) = x^4 + 1$$