

2. Let R be the region between the graph of $y = \cos(\pi x)$ and the x -axis for $0 \leq x \leq 1$.

a. Find the volume V of the solid obtained by rotating R about the x -axis.

$$\begin{aligned}
 V &= \pi \int_0^1 (\text{radius})^2 dx = \pi \int_0^1 |\cos(\pi x)|^2 dx = \pi \int_0^1 \cos^2(\pi x) dx \\
 &= \pi \int_0^1 \frac{1 + \cos(2\pi x)}{2} dx = \pi \left[\frac{1}{2}x + \frac{1}{4\pi} \sin(2\pi x) \right]_0^1 \\
 &= \pi \left(\frac{1}{2} + \frac{1}{4\pi} \underbrace{\sin(2\pi)}_0 - 0 - \frac{1}{4\pi} \underbrace{\sin(0)}_0 \right) = \frac{\pi}{2}
 \end{aligned}$$

b. Find the volume W of the solid obtained by rotating R about the y -axis.

$$\begin{aligned}
 W &= 2\pi \int_0^1 (\text{radius}) \cdot (\text{height}) dx = 2\pi \int_0^1 x \cdot |\cos(\pi x)| dx \\
 &= 2\pi \int_0^{1/2} x \cdot \cos(\pi x) dx + 2\pi \int_{1/2}^1 x \cdot (-\cos(\pi x)) dx \\
 &= 2\pi \left[\frac{1}{\pi} x \sin(\pi x) + \frac{1}{\pi^2} \cos(\pi x) \right]_0^{1/2} - 2\pi \left[\frac{1}{\pi} x \sin(\pi x) + \frac{1}{\pi^2} \cos(\pi x) \right]_{1/2}^1 \\
 &= 2\pi \cdot \left(\frac{1}{2\pi} \underbrace{\sin\left(\frac{\pi}{2}\right)}_1 + \frac{1}{\pi^2} \underbrace{\cos\left(\frac{\pi}{2}\right)}_0 - 0 - \frac{1}{\pi^2} \underbrace{\cos(0)}_1 \right) \\
 &\quad - 2\pi \cdot \left(\frac{1}{\pi} \underbrace{\sin(\pi)}_0 + \frac{1}{\pi^2} \underbrace{\cos(\pi)}_{-1} - \frac{1}{2\pi} \underbrace{\sin\left(\frac{\pi}{2}\right)}_1 - \frac{1}{\pi^2} \underbrace{\cos\left(\frac{\pi}{2}\right)}_0 \right) \\
 &= 2\pi \left(\frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{1}{\pi^2} + \frac{1}{2\pi} \right) = 2
 \end{aligned}$$

$$\int x \cos(\pi x) dx = \frac{1}{\pi} x \sin(\pi x) - \frac{1}{\pi} \int \sin(\pi x) dx = \frac{1}{\pi} x \sin(\pi x) + \frac{1}{\pi^2} \cos(\pi x) + C$$

$$\begin{aligned}
 u = x &\Rightarrow du = dx \\
 dv = \cos(\pi x) dx &\Rightarrow v = \frac{1}{\pi} \sin(\pi x)
 \end{aligned}$$