

1. Evaluate the limit $\lim_{x \rightarrow 0} \frac{(1 - \sin x)e^{(e^x - 1)} - 1}{x^4}$.

$$\lim_{x \rightarrow 0} \frac{(1 - \sin x)e^{e^x - 1} - 1}{x^4} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{-\cos x \cdot e^{e^x - 1} + (1 - \sin x) \cdot e^{e^x - 1} \cdot e^x}{4x^3}$$

$$= \lim_{x \rightarrow 0} \frac{-\cos x + (1 - \sin x)e^x}{4x^3} \cdot \lim_{x \rightarrow 0} \frac{e^{e^x - 1}}{1}$$

$$\stackrel{\text{L'H}}{\downarrow} = \lim_{x \rightarrow 0} \frac{\sin x - \cos x e^x + (1 - \sin x)e^x}{12x^2}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x + (1 - \sin x - \cos x)e^x}{12x^2}$$

$$\stackrel{\text{L'H}}{\downarrow} = \lim_{x \rightarrow 0} \frac{\cos x + (-\cos x + \sin x)e^x + (1 - \sin x - \cos x)e^x}{24x}$$

$$= \lim_{x \rightarrow 0} \frac{\cos x + (1 - 2\cos x)e^x}{24x}$$

$$\stackrel{\text{L'H}}{\downarrow} = \lim_{x \rightarrow 0} \frac{-\sin x + 2\sin x e^x + (1 - 2\cos x)e^x}{24} = \frac{0 + 0 - 1}{24} = -\frac{1}{24}$$