

2. We want to design a flower bed in the shape of a half-disk with diameter $|AB| = 20$ m. This half-disk will be separated into three regions by the line segments $[AC]$ and $[BC]$, and roses, hydrangeas and peonies will be planted in these regions, as shown in the figure.

The costs of planting roses, hydrangeas and peonies are 20 €/m², 30 €/m², and 50 €/m², respectively. Find the costs of the most expensive and the least expensive flower beds we can design.

$$\text{Cost} = 20 \cdot \left(\frac{1}{2} \cdot 20 \cdot 10 \sin \theta \right) + 30 \cdot \left(\frac{1}{2} \cdot 10^2 \cdot (\pi - \theta) - \frac{1}{2} \cdot 10 \cdot 10 \sin \theta \right) + 50 \cdot \left(\frac{1}{2} \cdot 10^2 \cdot \theta - \frac{1}{2} \cdot 10 \cdot 10 \sin \theta \right)$$

Maximize/Minimize $\text{Cost} = \left(\frac{3\pi}{2} + \theta - 2 \sin \theta \right) \times 10^3$ for $0 \leq \theta \leq \pi$

Critical points: $\frac{d}{d\theta} \text{Cost} = 1 - 2 \cos \theta = 0 \Rightarrow \theta = \frac{\pi}{3} \Rightarrow \text{Cost} = \left(\frac{11\pi}{6} - \sqrt{3} \right) \times 10^3$

Endpoints: $\theta = 0 \Rightarrow \text{Cost} = \frac{3\pi}{2} \times 10^3$
 $\theta = \pi \Rightarrow \text{Cost} = \frac{5\pi}{2} \times 10^3$

Since $3\sqrt{3} > 5 > \pi$, we have $\frac{5\pi}{2} > \frac{3\pi}{2} > \frac{11\pi}{6} - \sqrt{3}$.

The most expensive flower bed costs $\frac{5\pi}{2} \times 10^3$ €

The least expensive flower bed costs $\left(\frac{11\pi}{6} - \sqrt{3} \right) \times 10^3$ €

