

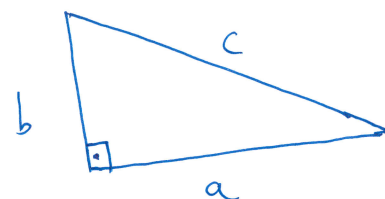
3. The lengths of the sides of a right triangle are changing as differentiable functions of time.

At a certain moment,

- the length of one of the legs is increasing at a rate of 1 cm/s,
- the length of the other leg is decreasing at a rate of 2 cm/s,
- the length of the hypotenuse is decreasing at a rate of 1 cm/s, and
- the area of the triangle is decreasing at a rate of $1/2$ cm²/s.

Find the length of the shortest side at this moment.

Let a, b, c be the lengths of the sides
and let A be the area of the triangle.



Then:

$$\frac{da}{dt} = 1 \text{ cm/s}, \quad \frac{db}{dt} = -2 \text{ cm/s}, \quad \frac{dc}{dt} = -1 \text{ cm/s}, \quad \frac{dA}{dt} = -\frac{1}{2} \text{ cm}^2/\text{s}$$

at our moment.

$$a^2 + b^2 = c^2 \xrightarrow{d/dt} 2a \frac{da}{dt} + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$$

$$\Downarrow$$

$$a - 2b = -\sqrt{a^2 + b^2} \quad (1)$$

$$A = \frac{1}{2}ab \xrightarrow{d/dt} \frac{1}{2}a \frac{db}{dt} + \frac{1}{2}b \frac{da}{dt} = \frac{dA}{dt}$$

$$\Downarrow$$

$$-2a + b = -1 \quad (2)$$

$$(1) \Rightarrow a^2 - 4ab + 4b^2 = a^2 + b^2 \Rightarrow 3b^2 = 4ab \Rightarrow b = 0 \text{ or } b = \frac{4}{3}a$$

$$b = 0 \xrightarrow{(2)} a = \frac{1}{2}, \text{ but } (a, b) = \left(\frac{1}{2}, 0\right) \text{ is not a solution of (1).}$$

$$b = \frac{4}{3}a \xrightarrow{(2)} a = \frac{3}{2} \xrightarrow{(2)} b = 2$$

The shortest side is $\frac{3}{2}$ cm long at this moment.