

2. Find $\left. \frac{d^2y}{dx^2} \right|_{(x,y)=(0,1)}$ if y is a differentiable function of x satisfying the equation $y^3 = y^2 \cos x + \sin x$.

$$y^3 = y^2 \cos x + \sin x$$

$\Downarrow d/dx$

$$3y^2 \cdot y' = 2y \cdot y' \cdot \cos x - y^2 \sin x + \cos x$$

$\searrow (x,y)=(0,1)$

$$3y' = 2y' - 0 + 1 \Rightarrow y' = 1 \text{ at } (x,y)=(0,1)$$

d/dx

$$6y \cdot (y')^2 + 3y^2 \cdot y'' = 2(y')^2 \cos x + 2y \cdot y'' \cos x - 2y \cdot y' \sin x - 2y \cdot y' \sin x - y^2 \cos x - \sin x$$

$\Downarrow (x,y)=(0,1), y'=1$

$$6 + 3y'' = 2 + 2y'' - 0 - 0 - 1 - 0$$

$\Downarrow$

$$y'' = -5 \text{ at } (x,y)=(0,1)$$