

1. Evaluate the following limits.

[Do not use L'Hôpital's Rule!]

$$\begin{aligned}
 \text{a. } \lim_{x \rightarrow 0^+} \frac{(x^{1/2} - x)^{1/2} - x^{1/4}}{x^{3/4}} &= \lim_{x \rightarrow 0^+} \frac{x^{1/4} \cdot (1 - x^{1/2})^{1/2} - x^{1/4}}{x^{3/4}} \\
 &= \lim_{x \rightarrow 0^+} \frac{(1 - x^{1/2})^{1/2} - 1}{x^{1/2}} = \lim_{x \rightarrow 0^+} \frac{((1 - x^{1/2})^{1/2} - 1) \cdot ((1 - x^{1/2})^{1/2} + 1)}{x^{1/2} \cdot ((1 - x^{1/2})^{1/2} + 1)} \\
 &= \lim_{x \rightarrow 0^+} \frac{1 - x^{1/2} - 1}{x^{1/2} \cdot ((1 - x^{1/2})^{1/2} + 1)} = - \lim_{x \rightarrow 0^+} \frac{x^{1/2}}{x^{1/2} \cdot ((1 - x^{1/2})^{1/2} + 1)} \\
 &= - \lim_{x \rightarrow 0^+} \frac{1}{(1 - x^{1/2})^{1/2} + 1} = - \frac{1}{1 + 1} = -\frac{1}{2}
 \end{aligned}$$

$$\text{b. } \lim_{x \rightarrow 0} \frac{\sin(x^2) \cos(\cot x)}{x}$$

$$\lim_{x \rightarrow 0} \frac{\sin(x^2)}{x} = \lim_{x \rightarrow 0} \left(\frac{\sin(x^2)}{x^2} \cdot x \right) = \lim_{x \rightarrow 0} \frac{\sin(x^2)}{x^2} \cdot \lim_{x \rightarrow 0} x = 1 \cdot 0 = 0$$

$$|\cos(\cot x)| \leq 1 \quad \text{for } 0 < |x| < \pi$$

$$\Downarrow$$

$$\left| \frac{\sin(x^2)}{x} \cdot \cos(\cot x) \right| \leq \left| \frac{\sin(x^2)}{x} \right| \quad \text{for } 0 < |x| < \pi$$

$$\Downarrow$$

$$- \left| \frac{\sin(x^2)}{x} \right| \leq \frac{\sin(x^2) \cos(\cot x)}{x} \leq \left| \frac{\sin(x^2)}{x} \right| \quad \text{for } 0 < |x| < \pi$$

Since $\lim_{x \rightarrow 0} \left| \frac{\sin(x^2)}{x} \right| = 0 = \lim_{x \rightarrow 0} \left(- \left| \frac{\sin(x^2)}{x} \right| \right)$, by Squeeze Theorem

$$\lim_{x \rightarrow 0} \frac{\sin(x^2) \cos(\cot x)}{x} = 0$$