

4. In each of the following, if the given statement is true for all f , then mark the $\square$ to the left of TRUE with a $\times$; otherwise, mark the $\square$ to the left of FALSE with a $\times$ and give a counterexample.

a. If f is one-to-one and differentiable on $(-\infty, \infty)$, then f^{-1} is differentiable.

☐ TRUE

☒ FALSE, because it does not hold for $f(x) =$

$$x^3$$

b. If f is differentiable on $(-\infty, \infty)$, then f has an antiderivative on $(-\infty, \infty)$.

☒ TRUE

☐ FALSE, because it does not hold for $f(x) =$

c. If f has an antiderivative on $(-\infty, \infty)$, then f is differentiable on $(-\infty, \infty)$.

☐ TRUE

☒ FALSE, because it does not hold for $f(x) =$

$$x^{1/3}$$

d. If $\int_0^\infty f(x) dx = 1$, then $g(x) = \int_0^\infty f(t/x) dt$ is increasing on $(0, \infty)$.

☒ TRUE

☐ FALSE, because it does not hold for $f(x) =$

e. If f is continuous and $f(x + 2\pi) = f(x)$ for all x , then $\int_0^{x+2\pi} f(t) dt = \int_0^x f(t) dt$ for all x .

☐ TRUE

☒ FALSE, because it does not hold for $f(x) =$

$$1$$