

3. Consider the function $f(x) = \exp((\ln x)^3) = e^{(\ln x)^3}$.

a. Compute $f'(x)$.

$$f'(x) = e^{(\ln x)^3} \cdot 3(\ln x)^2 \cdot \frac{1}{x}$$

b. Compute $f''(x)$.

$$f''(x) = e^{(\ln x)^3} \cdot 9(\ln x) \cdot \frac{1}{x^2} + e^{(\ln x)^3} \cdot 3 \cdot \left(2 \ln x \cdot \frac{1}{x^2} + (\ln x)^2 \cdot \left(-\frac{1}{x^2}\right) \right)$$

c. Sketch the graph of $y = f(x)$.

$$f'(x) = 0 \Rightarrow x = 1$$

$$f''(x) = e^{(\ln x)^3} \cdot 3 \cdot \frac{1}{x^2} \cdot \ln x \cdot (3(\ln x)^3 - \ln x + 2) = 0 \Rightarrow x = 1 \text{ or } x = e^{-1}$$

$$(\ln x + 1)(3(\ln x)^2 - 3\ln x + 2)$$

x	0	e^{-1}	1
f'	///	+	+
f''	///	+	-
f	///	+	+

$$f(1) = 1, \quad f(e^{-1}) = e^{-1}$$

$$\lim_{x \rightarrow 0^+} f(x) = 0$$

