

2a. Evaluate the limit $\lim_{x \rightarrow 0} \frac{1 + \ln(1+x) - \sqrt{2} \sin(x + \pi/4)}{x^3}$.

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{1 + \ln(1+x) - \sqrt{2} \sin(x + \pi/4)}{x^3} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{1+x} - \sqrt{2} \cos(x + \pi/4)}{3x^2} \\
 & \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{-\frac{1}{(1+x)^2} + \sqrt{2} \sin(x + \pi/4)}{6x} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{\frac{2}{(1+x)^3} + \sqrt{2} \cos(x + \pi/4)}{6} \\
 & = \frac{2 + \sqrt{2} \cdot \frac{1}{\sqrt{2}}}{6} = \frac{3}{6} = \frac{1}{2}
 \end{aligned}$$

2b. Evaluate the integral $\int_0^\infty \frac{dx}{(x^2+1)(3x^2+1)}$.

$$\begin{aligned}
 \int \frac{dx}{(x^2+1)(3x^2+1)} &= \int \frac{1}{2} \left(\frac{3}{3x^2+1} - \frac{1}{x^2+1} \right) dx = \frac{\sqrt{3}}{2} \arctan(\sqrt{3}x) - \frac{1}{2} \arctan(x) + C \\
 \int_0^\infty \frac{dx}{(x^2+1)(3x^2+1)} &= \lim_{c \rightarrow \infty} \int_0^c \frac{dx}{(x^2+1)(3x^2+1)} = \lim_{c \rightarrow \infty} \left[\frac{\sqrt{3}}{2} \arctan(\sqrt{3}x) - \frac{1}{2} \arctan(x) \right]_0^c \\
 &= \lim_{c \rightarrow \infty} \left(\frac{\sqrt{3}}{2} \arctan(\sqrt{3}c) - \frac{1}{2} \arctan(c) - \frac{\sqrt{3}}{2} \arctan(0) + \frac{1}{2} \arctan(0) \right) \\
 &= \frac{\sqrt{3}}{2} \cdot \frac{\pi}{2} - \frac{1}{2} \cdot \frac{\pi}{2} = (\sqrt{3}-1) \cdot \frac{\pi}{4}
 \end{aligned}$$