2. Determine whether each of the following series converges or diverges.

a. $\sum_{n=0}^{\infty} (\sqrt{n+1} - \sqrt{n})$

$$S_n = \sum_{k=0}^{n} (\sqrt{k+1} - \sqrt{k}) = (\sqrt{1} - \sqrt{0}) + (\sqrt{2} - \sqrt{1}) + \cdots + (\sqrt{n+1} - \sqrt{n}) = \sqrt{n+1}$$

$$\lim_{n \to \infty} S_n = \lim_{n \to \infty} \sqrt{n+1} = \infty$$

$$\Rightarrow \sum_{n=0}^{\infty} (\sqrt{n+1} - \sqrt{n}) \text{ diverges.}$$

b. $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}}$

$$\frac{1}{\sqrt{n+1} + \sqrt{n}} = \frac{\sqrt{n+1} - \sqrt{n}}{(\sqrt{n+1} + \sqrt{n})(\sqrt{n+1} - \sqrt{n})} = \sqrt{n+1} - \sqrt{n}$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} \text{ diverges by } \underline{\text{Part a}}.$$

c. $\sum_{n=0}^{\infty} (\sqrt{n+1} - \sqrt{n})^2$

$$c = \lim_{n \to \infty} \frac{(\sqrt{n+1} - \sqrt{n})^2}{\frac{1}{n}} = \lim_{n \to \infty} \frac{\left(\frac{1}{\sqrt{n+1} + \sqrt{n}}\right)^2}{\frac{1}{n}} = \left(\lim_{n \to \infty} \frac{1}{\sqrt{1+\frac{1}{n}} + 1}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Since $c = \frac{1}{4} > 0$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (the harmonic series),

$\sum_{n=0}^{\infty} (\sqrt{n+1} - \sqrt{n})^2$ diverges by LCT.